the only kind which can be collected unless annual or compound is called for in the contract. The courts have decided, probably because compound interest piles up so rapidly, that borrowers shall not be compelled to pay it unless they have agreed to. It is evident that if simple interest is unfair to a lender, so likewise is its analogue in discount, viz., true discount. In bank discount the bank not only collects a certain rate of interest on the loan, but also *on its own pay*, which is theoretically an absurd proceeding.

In partial payments we find the same difficulties presenting themselves. If payments are made within a year, according to the United States rule interest is collected and set at interest before it is due. The old so-called Connecticut rule tried to avoid this unfairness, but in so doing became too complicated for general use. Business men see that the mercantile rule is the only one which is fair to both parties when the whole transactions falls within a year. But the mercantile rule works injustice if the period covers more than a year, since by it the lender gets only simple interest, whereas the United States rule gives him a form of compound interest. The United States rule works injustice to the borrower whenever he pays less than the interest. Thus it is evident that the element of time and certain practical considerations have a great deal to do towards determining the appropriate method of counting interest.

Looked at from a practical standpoint it is easy to see why the banker collects bank instead of true discount. True discount requires a long division after a preliminary calculation. Tables could not be made for computing true discount which would be at all convenient to use. The great bulk of the loans made by banks are for less than 3 mouths. The difference between the bank and true discount of say $500 for 30 days is only a little over one cent. It would be worth more than 5 cents of the cashier's time to make the longer calculation. Of course the difference would not be so slight in every case, but it should be noted that one and sometimes more than one other person's time besides the cashier's is involved. Then liability to error is much greater in the longer calculation, and this is an important item. Hence it is plain that the method of discounting notes pursued by the banks was not adopted (as the writer once thought before he began making computations like the above) with the object of extorting more money from their customers, but for purely practical reasons.

## ALGEBRA.

**92. Proposed by W. F. BRADBURY, A. M., Head Master Latin School, Cambridge, Mass.**

Find the sum to $n$ terms of $1+3^3+5^3+\ldots.$ [From Charles Smith's *Elementary Algebra*, page 403].

**I. Solution by DR. E. D. ROE, Jr., Associate Professor of Mathematics in Oberlin College, P. O., Norwood, Mass.**

We have $s_{3,\,n}=1^3+2^3+\ldots.n^3=\left(\frac{n\,(n+1)}{2}\right)^2.$ (Todhunter's *Algebra*, page 263).

Now $s_{3,2n}=2^3 s_{3,n}+[1^3+3^3+5^3+\dots.(2n-1)^3]$.
Hence $1^3+3^3+5^3+\dots.(2n-1)^3=s_{3,2n}-8s_{3,n}=n^2(2n^2-1)$.

II. **Solution by J. OWEN MAHONEY, B. E., M. Sc., Master and Instructor in Mathematics and Science, Cooper Training School, Carthage, Tex.; COOPER D. SCHMITT, M. A., Professor of Mathematics, University of Tennessee, Knoxville, Tenn.; and J. SCHEFFER, A. M., Hagerstown, Md.**

Let $1^3+3^3+5^3+\dots.(2N-1)^3=A+BN+CN^2+DN^3+EN^4$.
Then $(2N+1)^3=B+(2N+1)C+D(3N^2+3N+1)+E(4N^3+6N^2+4N+1)$.
Equating coefficients, we find $E=2$, $D=0$, $C=-1$, $B=0$.
Therefore $S=A-N^2+2N^4$.
But when $N=1$, $A=0$. $\therefore S=N^2(2N^2-1)$.

III. **Solution by T. W. PALMER, A. M., Professor of Mathematics, University of Alabama, University, Alabama; G. B. M. ZEER, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.; CHARLES C. CROSS, Libertytown, Md.; J. SCHEFFER, A. M., Hagerstown, Md.; and ELMER SCHUYLER, A. M., High Bridge, N. J.**

The general term of the series is $(2n-1)^3$.

$$\therefore \Sigma(2n-1)^3=8\Sigma n^3-12\Sigma n^2+6\Sigma n-\Sigma 1$$

$$=8\frac{n^2(n+1)^2}{4}-12\frac{n(n+1)(2n+1)}{6}+6\frac{n(n+1)}{2}-n=n^2(2n^2-1).$$

IV. **Solution by J. SCHEFFER, A. M., Hagerstown, Md., and ELMER SCHUYLER, High Bridge, N. J.**

From the well known formula $1^3+2^3+3^3+\dots.n^3=\frac{n^2(n+1)^2}{4}$ we have

$$1^3+2^3+3^3+4^3+\dots.(2n-2)^3+(2n-1)^3=(2n-1)^2n^2.$$

Denoting the required sum by $S$, we have,

$S+2^3+4^3+6^3+\dots.(2n-2)^3=(2n-1)^2n^2$,

or $S+2^3[1+2^3+3^3+\dots.(n-1)^3]=(2n-1)^2n^2$,

or $S+2^3\frac{(n-1)^2n^2}{4}=(2n-1)^2n^2$, whence $S=n^2(2n^2-1)$.

V. **Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.; H. C. WHITAKER, A. M., Ph. D., Professor of Mathematics, Manual Training School, Philadelphia, Pa.; O. S. WESTCOTT, A. M., North Division High School, Chicago, Ill.; ELMER SCHUYLER, High Bridge, N. J.; A.H.BELL, Hillsboro, Ill.; JOSIAH H. DRUMMOND, LL. D., Portland, Me.; P. S. BERG, Principal of Schools, Larimore, N. D.; W. L. HARVEY, Portland, Me.; CHARLES E. MEYERS, Canton, O.; ALOIS F. KOVARIK, Decorah Institute, Decorah, Ia.; HAROLD C. FISKE, Union College, Schenectady, N. Y.; and J. SCHEFFER, A. M., Hagerstown, Md.**

Solving by the Differential Method, the general formula for the sum of the $n$ terms of a series is

$$Sum=na+\frac{n(n-1)}{2}a_1+\frac{n(n-1)(n-2)}{2\times3}a_2+\frac{n(n-1)(n-2)(n-3)}{2\times3\times4}a_3+\dots\dots$$

in which $a$=first term of series, and $a_1$, $a_2$, $a_3$, $a_4$, etc., are the respective first terms of the successive orders of differences.

Then 1    27    125    343    729....$(2n-1)^3$=given series.

26    98    218    386..................=first order of differences.

72    120    168......................=second order of differences.

48    48........................=third order of differences.

0..........................=fourth order of differences.

Therefore $a=1$, $a_1=26$, $a_2=72$, $a_3=48$, $a_4=0$.

Substituting these values in the general formula, we obtain

$$Sum=n+13n(n-1)+12n(n-1)(n-2)+2n(n-1)(n-2)(n-3)$$
$$=2n^4-n^2=n^2(2n^2-1)$$

=*the sum of the cubes of the first n odd numbers.*

When $n=4$, $n^2(2n^2-1)=496$.

When $n=5$, $n^2(2n^2-1)=1225=\square$.

COROLLARIES BY M. A. GRUBER.

COROLLARY 1. In a similar manner we find $2^3+4^3+6^3....+(2n)^3=2n^2(n+1)^2$=*the sum of the cubes of the first n even numbers.*

COROLLARY 2. By a similar process we obtain $1+2^3+3^3+4^3+....+n^3=\left[\frac{n(n+1)}{2}\right]^2$; or *the sum of the cubes of the first n natural numbers is equal to the square of the sum of the numbers.* For $1+2+3+4+....+n=\frac{n(n+1)}{2}$.

COROLLARY 3. $1+3^3+5^3+...+(2n-1)^3=n^2(2n^2-1)=\square$, when $n=1$, 5, 29, 169, 985, 5741, etc., or when $n$=the integral hypothenuse of a right triangle whose legs are consecutive integers. [See AMERICAN MATHEMATICAL MONTHLY, Vol. IV., No. 1, pages 24-27].

COROLLARY 4. $2^3+4^3+6^3+....+(2n)^3$ can never be a square; for $2n^2(n+1)^2$ is *twice a square.*

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## GEOMETRY.

109. Proposed by CHARLES CARROLL CROSS, Libertytown, Md.

Two circles, radii in ratio 3:1, centers $A$ and $O_1$ respectively, are drawn tangent externally to each other and internally to a given circle $O$, and on the same diameter; $O_2$ and $O_2'$ are drawn tangent externally to $O$ and internally to $A$ and $O_1$; $O_3$ and $O_3'$ are drawn tangent internally to $O$ and externally to $A$ and $O_2$; $O_3$ and $O_3'$ are drawn tangent internally to $O$ and externally to $A$ and $O_2$, $A$ and $O_2'$, respectively, and so on. Prove $O_4$. $O$, $O_4'$; $O_5$, $A$, $O_5'$; $O_9$, $A$, $O_3'$ and $O_{10}$, $O$, $O_2'$ are collinear. [The letters apply to the centers of the circles].

I. Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.

Construct the figure as indicated by the problem, and let $B$ be the point of tangency of the circles $A$ and $O$.